Modeling of wind-blown sand using cellular automata

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It is shown that our cellular automata models of wind-blown sand have the property of segregation in the heavier sand grain. This property, generally called particle size segregation, is a phenomenon in which heavy (coarse) grain accumulates on the crest of the sand ripple and light (fine) grain accumulates on the trough of the sand ripple. From the calculation of the quantity called total potential energy, which provides us with an effective measure for the extent of particle size segregation, we can conclude that the difference of creep dynamics (internal friction) between the heavy and the light grain is more effective on the segregation phenomena. And the present model is found to be an accurate tool for the analysis of this very peculiar phenomenon in the formation process of sand ripples in nature.

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At the beach or in places like a desert, we can see beautiful patterns of sand sheets. Such patterns, called sand ripples or sand dunes, are the results of the movement of a lot of sand grain. Sand ripples and dunes are different contexts of patterns. The main difference between them is the length scale. The length scale of a sand ripple is about 10 cm, and that of a dune is more than 10 m. In this paper, we mainly treat the formation of sand ripples, where one can see some characteristic features or phenomena, such as, e.g., a proportional relation between wind velocity and wavelength of the sand ripple, and the existence of a threshold of wind velocity in the formation of sand ripples. We have studied of these phenomena previously [1,2]. Here we will concentrate on a phenomenon called particle size segregation, i.e., heavy (coarse) grain accumulates on the crest of the sand ripple and light (fine) grain accumulates on the trough of the sand ripple [3]. This is a phenomenon peculiar to the formation of sand ripples [4].

The dynamics of wind-blown sand can be classified into three types [5], suspension, saltation, and surface creep. Saltation is a dynamics such that sand grain jumps to another place in a short time. Surface creep is a rolling process on the surface of a sand sheet. Suspension is a flight dynamics of very fine grains, e.g., the dynamics of volcanic ashes, so there is no direct effect on the formation of local patterns such as sand ripples. Therefore we can ignore the suspension dynamics.

If we take into account all of the dynamics of the wind-blown sand, we have to solve equations which have many degrees of freedom, but the complexity of the model often prevents us from discovering the simple principle. So we use two dimensional (lateral plus height) cellular automata (CAs) to model the wind-blown sand. CAs are dynamical systems in which state, time, and space are all discrete, and which have recently been used for studying many physical and chemical phenomena [6]. Our CAs have three states $S^t_{i,j}$, ϕ_0 [nothing (air)], ϕ_1 (light), and ϕ_2 (heavy) ($\phi_1 < \phi_2$),

$$S^t_{i,j} = \{\phi_0, \phi_1, \phi_2\}, \tag{1}$$

where i is the lateral index, j is the height index, t is time, and the different states represent the different grain sizes. On modeling the wind-blown sand, we take into account dynamics which comes mainly from a weight of the grain. Of course it is well known that the role of the volume difference is very important in the segregation phenomena of the granular material, but in the case of sand-ripple formation, unlike the general granular flow [7] the moving grain is only at the surface of the sand ripple so we can think that there is a little convective effect (volume effect) on the segregation of the wind-blown sand. In this model, we assume the wind direction to be uniform. Each state evolves by following two procedures.

The first accounts for the saltation dynamics, where we use the same relation for the flight length as in previous works [1,2]. The flight length L is a linear function of $h(i)$,

$$L = L_0 + ah(i), \tag{2}$$

where $h(i)$ is the height of the sand ripple at position i defined by

$$h(i) = \sum_{j=1}^{h_{max}} \sigma S^t_{i,j}, \tag{3}$$

and

$$\sigma = \begin{cases} 0, & S^t_{i,j} = \phi_0 \\ 1/S^t_{i,j}, & S^t_{i,j} = \phi_1, \phi_2 \end{cases} \tag{4}$$

where h_{max} is the vertical system size. Generally, the mass of the grain is proportional to the radius cubed, so we can define the "mass" of a grain as

$$m_{i,j} = \phi_n^3, \qquad n = 1, 2. \tag{5}$$

The coefficient a is related to $m_{i,j}$ as

$$a = a'/m_{i,j}, \tag{6}$$

because the flight length of a light grain is longer than the flight length of a heavy grain. a' is proportional to the wind velocity, so the stronger the wind, the longer the flight length L. In this simulation, we set $L_0 = 0$. The exchange of state by saltation is

$$\begin{aligned} S^{t'}_{i,h(i)} &= \phi_0, \\ S^{t'}_{i+L,h(i+L)+1} &= S^{t}_{i,h(i)}, \\ h(i) &= h(i) - 1, \\ h(i+L) &= h(i+L) + 1, \end{aligned} \tag{7}$$

where t' is the intermediate time step. Every grain at the surface of the pile saltates every time step.

The second procedure is the surface creep dynamics. We use the gradient of the height,

$$h(i) - h(i+1) \equiv \theta, \tag{8}$$

for the surface creep dynamics. If θ exceeds some critical value h_c, then the upper grain falls to the neighboring site,

$$\begin{aligned} S^{t+1}_{i_{out},h(i_{out})} &= \phi_0, \\ S^{t+1}_{i_{in},h(i_{in})+1} &= S^{t'}_{i_{out},h(i_{out})}, \\ h(i_{out}) &= h(i_{out}) - 1, \\ h(i_{in}) &= h(i_{in}) + 1, \end{aligned} \tag{9}$$

where we take $i_{out} = i$ and $i_{in} = i + 1$ for $\theta > 0$, and $i_{out} = i + 1$ and $i_{in} = i$ for $\theta < 0$. There are two types of h_c. If the moving grain is "light," i.e., state ϕ_1, then h_c is given by h_{1c} and in the opposite case h_c is given by h_{2c}, i.e.,

$$h_c = \begin{cases} h_{1c} & \text{if } S_{i_{out},h(i_{out})} = \phi_1 \\ h_{2c} & \text{if } S_{i_{out},h(i_{out})} = \phi_2. \end{cases} \tag{10}$$

In general, a heavy grain is more stable against some external perturbation than a light grain, so we set $h_{2c} > h_{1c}$. This argument corresponds to the mass difference and friction difference of each grain. The saltation is a one-directional movement, but creep occurs in both directions. Figure 1 gives a schematic explanation of the dynamics.

Before explaining the results of the simulation, we introduce a total potential energy of the system. The total potential energy (TPE) is the sum of the potential energy of each grain and is defined by

$$U = \sum_i \sum_j m_{i,j} g j, \tag{11}$$

where g is a gravitational constant. The TPE is a quantity that characterizes the arrangement of heavy grain and light grain [8]. Clearly, if heavy grain accumulates on higher positions than the initial positions, the TPE increases. Furthermore, the TPE measures the "roughness" of the surface of the system.

Let ρ be a mass density of the system, and g a gravitational constant. Then the TPE of the system can be written in a continuous form as

$$U = \int_0^N \int_0^{h(x)} \rho(x,y) g y \, dy \, dx, \tag{12}$$

where $h(x)$ is some function for the land form, and x is the horizontal position whose maximum value is the system size N. Assuming that

$$\rho(x,y) = \rho = \text{const}, \tag{13}$$

then

$$\begin{aligned} U &= \rho g \int_0^N \int_0^{h(x)} y \, dy \, dx \\ &= \frac{1}{2} \rho g \int_0^N h^2(x) dx. \end{aligned} \tag{14}$$

If we decompose $h(x)$ into the average height $\bar{h}$ and the fluctuation $\delta h(x)$,

$$h(x) = \bar{h} + \delta h(x), \tag{15}$$

and assume the total height of the system (total mass) to be conserved,

$$\int_0^N \delta h(x) dx = 0, \tag{16}$$

then the variation of the TPE between a flat surface and a rough surface is given by

$$\begin{aligned} \delta U &= U_{rough} - U_{flat} \\ &= \frac{1}{2} \rho g \left[\int_0^N \left[\bar{h} + \delta h(x) \right]^2 dx - \int_0^N \bar{h}^2 dx \right] \\ &= \frac{1}{2} \rho g \int_0^N (\delta h)^2 dx \geq 0. \end{aligned} \tag{17}$$

δh is the "roughness" of the system, so the rougher the surface, the larger the TPE. But in the case of $\rho \neq$const, the TPE may decrease below the TPE of a flat surface even if the roughness of the system increases, i.e., most of the heavy components sink to lower heights than that of the initial state. This argument can easily be extended to higher dimensional systems and more general surface roughening processes against the direction of the force.

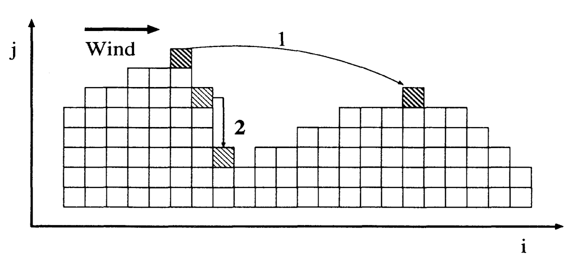

FIG. 1. Schematic representation of the dynamics. 1: saltation dynamics, 2: creep dynamics.

We use $\Delta U_{seg} = U -$ (TPE as $\rho = \bar{\rho}$) as the index of particle size segregation, where $\bar{\rho}$ is an averaged value of the heavy and light grain.

The simulation starts with a random initial condition

$$h_{init} = h_0 + \delta, \tag{18}$$

where the random number δ is very small compared to the average height h_0. The boundary of the top is free, the boundaries of the bottom are fixed, and those of the left and right are periodic. In this simulation, we set the ratio of coarse and fine grains as 2:8. Initially, the two types of grains are distributed randomly. The dynamics of a grain is deterministic, i.e., once an initial state is given then the evolution of the system is determined for eternity.

Our CA model can be classified into four types depending on whether the system has differences between heavy and light grain dynamics or not.

	Saltation difference	Creep difference
Type 1	YES	YES
Type 2	NO	YES
Type 3	YES	NO
Type 4	NO	NO

For example, Type 2 has the same saltation dynamics, but different creep dynamics for heavy and light grain. We simulate these four types to study which dynamics is the most effective for obtaining particle size segregation. The results of the calculation of the ΔU_{seg} for each type are drawn in Fig. 2. The largest increase in the ΔU_{seg} occurred for type 2. A line of type 4's ΔU_{seg} can be thought of as a criterion for particle size segregation. Consequently, if the ΔU_{seg} is greater than the ΔU_{seg} of type 4 we can think that the system has the property of particle size segregation. But if a ΔU_{seg} is smaller than the type 4 then the system also has a property of particle size segregation but the reverse property in contrast to the former one. A distinct difference can be seen between ΔU_{seg} of each type.

Here, we think about the ratio of ϕ_1's flight length L_1 to ϕ_2's flight length L_2,

$$\frac{L_1}{L_2} = \frac{\phi_2^3}{\phi_1^3} = \gamma. \tag{19}$$

We tested some values of γ but no significant change has been seen in ΔU_{seg} of each type.

Next, we examine the vertical grain distribution rates (percentage)

$$\omega_n(j) = \frac{\sum_i N\phi_n}{\sum_{n=1,2} \sum_i N\phi_n} \times 100, \tag{20}$$

where $N\phi_n$ means number of ϕ_n at j and i lateral, j vertical index. Figure 3 gives the grain distributions of heavy grain (ϕ_2) for each type. From Fig. 3(b), we can see that heavy grain accumulates on higher positions than the initial positions in type 2, but the nature of type 3 is contrary to that of type 2.

From these results, we can conclude that the most effective dynamics for particle size segregation is the difference in creep dynamics, i.e., the difficulty of movement. This difficulty comes from friction or the inertia of the grain. This inner friction is the difference between the real fluid and the sand grains. But the opposite type of particle size segregation can occur when the particle has no difference in creep dynamics but has difference in saltation dynamics. That is to say, if there is no difference in creep dynamics then the heavy grain has a tendency to go to a lower position. This is quite natural in a real fluid like a binary fluid. This is because the system has a tendency to select one wavelength depending on the average flight length of sand grains [1,2], but if there exist two average flight lengths then fluidlike mixing between crests and troughs will occur.

Next, we study how wavelength depends on the wind velocity. In the previous works [1,2], our simulation showed a linear dependence of the wavelength of a ripple on the wind velocity. Figure 4 is a plot of a' (proportional

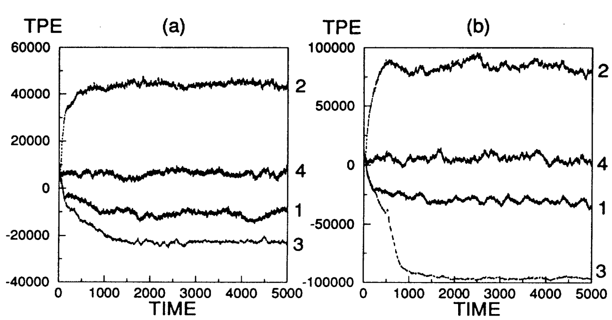

FIG. 2. Time evolution of the ΔU_{seg} of each type. The numbers at the right side show the model types of the lines. (a) is $a' = 1.6$ and (b) is $a' = 2.0$. In both cases $\gamma = 8, h_{1c} = 1, h_{2c} = 2, \phi_1 = 1, \phi_2 = 2$, and the system size is 256×128.

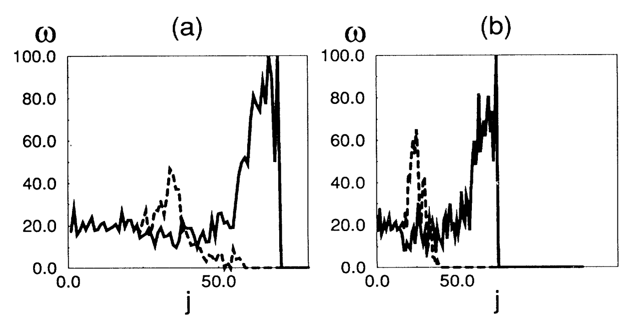

FIG. 3. Plots of $\omega_2(j)$ for type 2 and type 3. The solid lines in Fig. 3 are in the case of type 2, and the dashed lines are in the case of type 3. (a) and (b) show $a' = 1.6$ and $a' = 2.0$, respectively. The model parameters are the same as in Fig. 2.

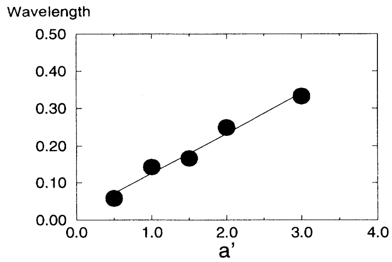

FIG. 4. The plots of a' vs normalized wavelength. In this case $\gamma = 8$, $h_{1c} = 1$, $h_{2c} = 2$, $\phi_1 = 1$, $\phi_2 = 2$, and the system size is 256×128.

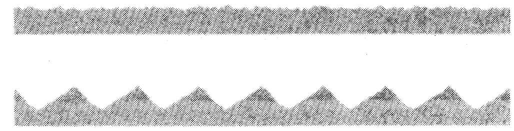

FIG. 5. Snapshot of the simulation. The upper part of the figure is the initial state and the lower part of the figure is the fully developed state. Dark colored cells are ϕ_2 and light colored cells are ϕ_1. The wind direction is left to right. The ripple profile appears asymmetric. The system size is 512×200. $a' = 1.2, h_{1c} = 1, h_{2c} = 2, \phi_1 = 1, \phi_2 = 2$.

wind velocity) vs normalized wavelength of the ripple. The wavelength increases proportionally with the wind velocity. Each type has the same property. Figure 5 is a snapshot of the simulation. The profile of the system looks asymmetrical, i.e., the downwind slope is steeper than the upwind slope.

To conclude, differences in the TPE of each type show that the most effective dynamics for particle size segregation are obtained by a difference of the grain stability against external force. The crest of the ripple is the most unstable place for the grain, so the most stable grains (heavy grains) can keep themselves at the crest of the sand ripple. Owing to this effect, sand grains are sorted by their size (stability), that is to say, the grain is sorted from the most unstable place (crest) to the most stable place by its stability. In this way, the phenomenon of particle size segregation emerges.

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